

Inflationary Affleck-Dine Scalar Dynamics and Isocurvature Perturbations

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Abstract

We consider the evolution of the Affleck-Dine scalar during D-term and F-term inflation and solve the combined slow-roll equations of motion. We show that for a typical case, where both the Affleck-Dine scalar and inflaton initially have large values, in D-term inflation the Affleck-Dine scalar is driven to a fixed value, with only a very slight dependence on the number of e-foldings. As a result, there is a definite prediction for the ratio of the baryonic isocurvature perturbation to the adiabatic perturbation. In minimal (d=4) Affleck-Dine baryogenesis the relative isocurvature contribution to the CMB angular power spectrum amplitude is predicted to be in the range $0.01 - 0.1$, which can account for present large-scale structure observations and should be observable by PLANCK. In a very general case, scale-invariance of the adiabatic perturbations from the Affleck-Dine scalar imposes a lower bound of about 0.01 for d=4. For

d=6 the isocurvature perturbation may just be observable, although this is less certain. We also consider F-term inflation and show that the magnitude of the baryonic isocurvature perturbation is fixed by the value of H during inflation. For typical values of H the isocurvature perturbation could be close to present observational limits.

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1 Introduction

With a detailed study of the cosmic microwave background (CMB) planned over the coming decade [1], it is important to consider the possible implications for particle physics models. The interaction of the particle physics model with the model of inflation may generate a CMB which deviates from that expected on the basis of inflation alone. We have previously discussed such a case [2, 3], the Minimal Supersymmetric Standard Model (MSSM) with Affleck-Dine baryogenesis [4, 5] in the context of D-term inflation [6, 7]. Affleck-Dine baryogenesis is a very natural and effective candidate for the origin of the baryon asymmetry in SUSY models, in particular in the MSSM, where it is the only known candidate in the absence of electroweak baryogenesis, for which only a small window of Higgs mass remains [8]. In the context of D-term inflation, the Affleck-Dine scalar provides a second source of adiabatic perturbations, and requiring that the deviation from scale-invariance due to the Affleck-Dine scalar is acceptably small imposes an upper bound on the magnitude of the Affleck-Dine scalar, which in turn translates into a lower bound on the isocurvature perturbations associated with quantum fluctuations of the phase of the Affleck-Dine field [2]. The spectrum of CMB perturbations thus provides a feasible testing bench for Affleck-Dine baryogenesis.

In this paper we wish to consider the dynamical evolution of the Affleck-Dine (AD) scalar during inflation more generally and in more detail. SUSY inflation models are broadly of two types, D-term or F-term, depending on the source of the vacuum energy driving inflation [7]. D-term inflation models have the advantage that the inflaton does not receive order H^2 corrections to its mass squared [6], which would prevent slow-rolling and produce a highly non-scale invariant spectrum of perturbations. Although D-term inflation models have the disadvantage that the inflaton field must start at values close to the Planck scale in order to provide sufficient inflation [7, 9, 10], which requires suppression of Planck-scale corrections to the potential, they nevertheless have become the favoured class of SUSY inflation models. F-term inflation models generically have dangerous order H^2 corrections to the inflaton mass squared [11] (and to the mass squared terms of all other scalars, in particular the AD scalar). However, these

corrections might be avoided for the inflaton as a result of accidental cancellations, a special choice of the superpotential and Kähler potential [12], or radiative corrections to the inflaton [13]. We do not, however, expect the cancellation to simultaneously apply to any other scalars, and so we expect that in the case of F-term inflation, unlike D-term inflation, the AD scalar will have an order H^2 correction to its mass squared term. Because of the different mass squared terms during inflation, the dynamics of the AD scalar in the two cases will be quite different, with correspondingly different consequences for the isocurvature perturbations.

The paper is organized as follows. In Section 2 we consider the case of D-term inflation. We first discuss the slow-rolling dynamics of the AD scalar and the inflaton. We then discuss the adiabatic perturbations, obtaining an upper bound on the magnitude of the AD scalar from scale-invariance of the adiabatic perturbations. We next discuss the isocurvature perturbations, predicting their magnitude for the case where the AD scalar and inflaton have initially large values and more generally obtaining a lower bound from the adiabatic perturbation upper bound. In Section 3 we consider the case of F-term inflation, showing that when the CMB perturbations leave the horizon the AD scalar is likely to be close to the minimum of its potential and that the magnitude of the isocurvature perturbations is then fixed by the value of H during inflation. In Section 4 we discuss our conclusions. In an Appendix we briefly review Affleck-Dine baryogenesis.

2 D-term Inflation

2.1 Slow-Roll Dynamics of the Affleck-Dine Scalar

D-term inflation [6] is a form of hybrid inflation [14], driven by the energy density of a Fayet-Iliopoulos D-term. The inflaton S is coupled to fields oppositely charged under a Fayet-Iliopoulos $U(1)_{FI}$ via the superpotential term

$$W = \lambda S \psi_+ \psi_- . \tag{1}$$

The tree-level scalar potential, including the $U(1)_{FI}$ D-term, is then

$$V = |\lambda|^2 \left(|\psi_+ \psi_-|^2 + |S\psi_+|^2 + |S\psi_-|^2 \right) + \frac{g^2}{2} \left(|\psi_+|^2 - |\psi_-|^2 + \xi^2 \right)^2, \quad (2)$$

where ξ^2 is the FI term and g is the $U(1)_{FI}$ coupling. The global minimum of the potential is at $S = 0$, $\psi_+ = 0$, $\psi_- = \xi$. However, for $S > S_{crit} \equiv g\xi/\lambda$, the minimum is at $\psi_+ = \psi_- = 0$ and there is a non-zero energy density $V_o = g^2\xi^4/2$. There will be an S potential, however, from 1-loop corrections. Thus for $S > S_c$ the inflaton potential is given by [6]

$$V(S) = V_o + \frac{g^4\xi^4}{32\pi^2} \ln \left(\frac{S^2}{Q^2} \right) \quad ; \quad V_o = \frac{g^2\xi^4}{2}, \quad (3)$$

where Q is a renormalization scale for the radiative correction. ξ is fixed by the observed CMB fluctuations [15] to be 6.6×10^{15} GeV [16]. The total number of e-foldings of inflation, N , remaining at a given value of S when the potential is dominated by the inflaton is related to S by

$$S = \frac{gN^{1/2}M}{2\pi}, \quad (4)$$

where $M = M_{Pl}/\sqrt{8\pi}$ is the mass scale of supergravity (SUGRA) corrections. The time when the observable CMB perturbations were formed corresponds to $N \approx 50$.

The scalar potential for the AD field $\Phi \equiv \phi e^{i\theta}/\sqrt{2}$ along an F- and D-flat direction of dimension d is given by

$$V(\phi) = \frac{\lambda^2 |\Phi|^{2(d-1)}}{M^{2(d-3)}}, \quad (5)$$

corresponding to a non-renormalizable superpotential term of the form $W = \lambda\Phi^d/dM^{d-3}$ lifting the flat direction. The coupling λ is unknown, but if the physical strength of the non-renormalizable interactions is set by the SUGRA scale M then we expect that $\lambda \approx 1/(d-1)!$ [9]. In practice, the superpotential term lifting the flat direction is also the B and CP violating operator responsible for AD baryogenesis, inducing a baryon asymmetry in the coherently oscillating ϕ condensate (see Appendix). For the case of R-parity conserving models, the B violating operators have even dimension, $d = 4, 6, \dots$. We will refer to the $d = 4$ case as minimal AD baryogenesis.

For large initial values of ϕ , $S \sim \mathcal{O}(M)$, the dynamics is first dominated by $V(\phi)$. For sufficiently large ϕ the effective mass squared of the ϕ field, $V''(\phi)$, becomes larger than H^2 . This occurs once $\phi > \phi_H$, where

$$\phi_H = \frac{2^{\frac{(d-1)}{2(d-2)}}}{(6(2d-2)(2d-3))^{\frac{1}{2(d-2)}}} \left(\frac{g}{\lambda}\right)^{\frac{1}{d-2}} \xi^{\frac{2}{d-2}} M^{\frac{d-4}{d-2}}. \quad (6)$$

If $\phi_i > \phi_H$, ϕ will initially rapidly oscillate in its potential, with an amplitude damped as $\phi \propto a^{-3/d}$, where a is the scale factor [17]. However, this period will end before the onset of inflaton domination and typically after less than 10 e-foldings of inflation. The system then enters the regime where both ϕ and S are slowly rolling.

The slow-rolling dynamics of the scalar fields is given by the solution of

$$3H\dot{\Psi}_a = -\frac{\partial V(\Psi_a)}{\partial \Psi_a} \quad ; \quad H = \left(\frac{\sum_a V(\Psi_a)}{3M^2}\right)^{1/2}, \quad (7)$$

where $\Psi_a \equiv S, \phi$. By taking the ratio of the equations for ϕ and S we obtain

$$\frac{\partial \phi}{\partial S} = \frac{16\pi^2(d-1)\lambda^2\phi^{(2d-3)}S}{2^{d-2}g^4\xi^4M^{2(d-3)}} , \quad (8)$$

which has the general solution

$$\phi = \phi_i \left[1 + \alpha_d \phi_i^{2d-4} (S_i^2 - S^2)\right]^{-1/(2d-4)} \quad ; \quad \alpha_d = \frac{16\pi^2(d-2)(d-1)\lambda^2}{2^{d-2}M^{2(d-3)}g^4\xi^4}, \quad (9)$$

where ϕ_i and S_i are the initial values at the onset of inflation. We observe two features of this solution. Firstly, since S_i is large compared with the value of S at $N = 50$, we see that for sufficiently large ϕ_i the value of ϕ at late times is *fixed* by S_i ,

$$\phi \equiv \phi_* \approx \left(\frac{1}{\alpha_d}\right)^{\frac{1}{2d-4}} \frac{1}{S_i^{1/(d-2)}}. \quad (10)$$

This is true if $\phi_i > \phi_*$, otherwise ϕ simply remains at ϕ_i . Secondly, we can relate S_i to the total number of e-foldings during the $V(S)$ dominated period of inflation. In general, for sufficiently large ϕ_i , we could have an initial period of $V(\phi)$ dominated inflation. We can show, however, that during this period S does not significantly change from S_i . The potential is dominated by $V(\phi)$ once $\phi > \phi_S$, where

$$\phi_S = \frac{\sqrt{2}M^{\frac{d-3}{d-1}}}{\lambda^{\frac{1}{d-1}}} \left(\frac{g^2\xi^4}{2}\right)^{\frac{1}{2(d-1)}}. \quad (11)$$

ϕ_S is generally less than ϕ_H , therefore ϕ will be slow-rolling during $V(S)$ domination.

From Eq. (9) we find that the condition for S to change significantly from S_i at a given value of ϕ is given by

$$S_i < \frac{1}{\alpha_d^{1/2}} \left(\frac{1}{\phi} \right)^{d-2}. \quad (12)$$

Thus the condition for S to change significantly during $V(\phi)$ dominated inflation is given by Eq. (12) with $\phi = \phi_S$,

$$S_i < S_{i_c} \approx \frac{2^{\frac{d-2}{2(d-1)}} g^{\frac{d}{d-1}} \xi^{\frac{2}{d-1}} M^{\frac{d-3}{d-1}}}{4\pi \lambda^{\frac{1}{d-1}}}. \quad (13)$$

Since S_{i_c} is small compared with M , whereas the value of S required to generate 50 e-foldings of inflation, $S_{50} = g\sqrt{50}M/(2\pi)$, is close to M , it follows that $S_i (> S_{50})$ will generally be larger than S_{i_c} and so the inflaton will remain at S_i until the Universe becomes inflaton dominated. In this case the total number of e-foldings of inflation during inflaton domination is given by N_S , where $S_i = (g/2\pi)N_S^{1/2}M$. Therefore, if $\phi_i > \phi_*$, ϕ at $N \approx 50$ will be given by

$$\phi_* \approx \left(\frac{1}{\alpha_d} \right)^{\frac{1}{2d-4}} \left(\frac{2\pi}{gMN_S^{1/2}} \right)^{\frac{1}{d-2}}. \quad (14)$$

The dependence on N_S is quite weak; for the case of $d = 4$ ($d = 6$) Affleck-Dine baryogenesis, $\phi_* \propto N_S^{-1/4}$ ($N_S^{-1/8}$). Thus if there is not an extremely large number of e-foldings of inflation during inflaton domination compared with the minimum $N \approx 50$ necessary for the flatness of the Universe (i.e. S is not very large compared with M), we can essentially fix the value of ϕ_* . In this case we will be able to *predict* the magnitude of the baryonic isocurvature perturbation.

It is interesting to speculate on the likely initial values of ϕ and S . The initial value of S is likely to be arbitrary in D-term inflation models, because the potential must be very flat even to values of the order of the Planck scale. This is because, as noted above, S_{50} is close to the Planck scale, in which case we expect Planck scale suppressed superpotential terms to become important. (This is the flatness problem of D-term inflation models [9, 10].) A flat potential can be maintained by imposing a symmetry on S (e.g. an R-symmetry [9, 18]) to prevent these dangerous Planck

suppressed terms, so eliminating any potential for S beyond the 1-loop logarithmic term. In this case there is no obvious energy density constraint on the initial value of S . For $V(\phi)$, the energy density rapidly increases as ϕ approaches M . We might then impose a "chaotic inflation" -type initial condition, $V(\phi_i) \approx M^4$ [19]. This would give

$$\phi_i \approx \frac{\sqrt{2}M}{\lambda^{\frac{1}{d-1}}} . \quad (15)$$

By directly solving the slow-roll equations for ϕ and S we can show that the total number of e-foldings of inflation is given by

$$N_T = N_\phi + N_S \approx \frac{1}{4(d-1)} \frac{\phi_i^2}{M^2} + \frac{4\pi^2 S_i^2}{g^2 M^2} , \quad (16)$$

where N_ϕ is the number of e-foldings during $V(\phi)$ domination if $\phi_i > \phi_S$. From this we see that the $V(S)$ dominated contribution to the total number of e-foldings will dominate if

$$N_S > \frac{1}{2(d-1)\lambda^{2/(d-1)}} . \quad (17)$$

Since $N_S > 50$, this will be satisfied so long as λ is not very small (for example, if $\lambda \approx 1/(d-1)!$). In this case the value of ϕ when the CMB perturbations are formed, which in turn fixes the magnitude of the isocurvature perturbation, will be determined by the *total* number of e-foldings of inflation, $N_T \approx N_S$.

2.2 Adiabatic Perturbations from the Affleck-Dine Scalar

The potential for the AD scalar is far from flat, and so if the magnitude of the AD scalar is large it will cause a large deviation of the adiabatic perturbation from scale-invariance. This will impose an upper limit on the magnitude of the AD scalar at 50 e-foldings.

The deviations from scale-invariance are characterized by the spectral index, defined so that the density perturbation of present wavenumber k is of the form $\delta\rho/\rho \propto k^{\frac{n-1}{2}}$ on re-entering the horizon, where [7]

$$n = 1 + 2\eta - 6\epsilon . \quad (18)$$

For the case of a single inflaton η and ϵ are given by the standard expressions [7, 20]

$$\eta = M^2 \frac{V_S''}{V_S} \quad (19)$$

and

$$\epsilon = \frac{M^2}{2} \left(\frac{V_S'}{V_S} \right)^2, \quad (20)$$

where $V_S = V(S)$, $V_S' = \partial V / \partial S$, In order to discuss the influence of the AD scalar, we must generalize these expressions to the case of two scalar fields. For a potential of the form $V = V(S) + V(\phi)$ we find that,

$$\eta = -\frac{M^2}{(V_S' + V_\phi')V} \left[V_S' V_S'' + V_\phi' V_\phi'' - \frac{2(V_S' + V_\phi')(V_S'' V_S'^2 + V_\phi'' V_\phi'^2)}{(V_S'^2 + V_\phi'^2)} \right] \quad (21)$$

and

$$\epsilon = \frac{M^2}{(V_S' + V_\phi')V} \left[\frac{(V_S' + V_\phi')(V_S'^2 + V_\phi'^2)}{2V} \right]. \quad (22)$$

For the case of D-term inflation, if $V_\phi' < V_S'$ and $V_\phi'' < V_S''$, we obtain the conventional results

$$\eta = -\frac{1}{2N} \quad (23)$$

and

$$\epsilon = \frac{g^2}{32\pi^2 N}. \quad (24)$$

(The main contribution to scale-dependence therefore comes from η .) In general, deviation from scale-invariance due to the AD scalar first arises when $V_\phi'' > V_S''$, with $V_\phi' \ll V_S'$ and $V_\phi \ll V_S$ being still satisfied. In this case we can expand η to obtain corrections to the conventional D-term inflation case,

$$\eta \approx M^2 \frac{V_S''}{V_S} - M^2 \frac{V_\phi' V_\phi''}{V_S V_S'}. \quad (25)$$

Thus the deviation of the spectral index from scale invariance due to the AD scalar is

$$\Delta n_\phi \approx -\frac{2V_\phi'' V_\phi' M^2}{V_S V_S'}. \quad (26)$$

Requiring that $|\Delta n_\phi| < K$ (present CMB observations imply that $n = 1.2 \pm 0.3$ [15]; in the following we will use $K < 0.2$ [7]) imposes an upper bound on ϕ ,

$$\phi < \phi_c = k_d \left(\frac{K}{\sqrt{N}} \right)^{\frac{1}{4d-7}} g^{\frac{5}{4d-7}} \lambda^{\frac{-4}{4d-7}} \xi^{\frac{8}{4d-7}} M^{\frac{4d-15}{4d-7}} , \quad (27)$$

where

$$k_d = \left(\frac{2^{2(d-1)}}{128\pi(d-1)^2(2d-3)} \right)^{\frac{1}{4d-7}} . \quad (28)$$

For the case of minimal $d = 4$ Affleck-Dine baryogenesis we obtain

$$\phi_c = 0.53 \left(\frac{K}{\sqrt{N}} \right)^{\frac{1}{9}} \left(g^5 \lambda^{-4} \xi^8 M \right)^{\frac{1}{9}} \sim 10^{16} \text{ GeV} , \quad (29)$$

whilst for $d = 6$ baryogenesis

$$\phi_c = 0.77 \left(\frac{K}{\sqrt{N}} \right)^{\frac{1}{17}} \left(g^5 \lambda^{-4} \xi^8 M^9 \right)^{\frac{1}{17}} \sim 10^{17} \text{ GeV} . \quad (30)$$

2.3 Isocurvature Perturbations from the Affleck-Dine Scalar

Isocurvature perturbations of the baryon number arise from the AD scalar if the angular direction is effectively massless (i.e. mass small compared with H) during and after inflation. The resulting perturbations will be unsuppressed until the baryon number forms. This in turn requires that there are no order H corrections to the SUSY-breaking A-terms. In the effective softly broken MSSM at scales $\ll M$, such A-terms can arise only from terms with linear couplings of the inflaton superfield to gauge-invariant operators of MSSM superfields ϕ_i , for example,

$$\frac{1}{M} \int d^2\theta S W_i + h.c. \sim \frac{F_S}{M} W_i + h.c. \quad (31)$$

and

$$\frac{1}{M} \int d^2\theta d^2\bar{\theta} S \phi_i^\dagger \phi_i + h.c. \sim \frac{F_S^\dagger F_{\phi_i} \phi_i}{M} + h.c. \quad (32)$$

In the case of D-term inflation, the inflaton cannot induce an A-term either during or after inflation, since $F_S = 0$ in general [9]. More generally, if there is a symmetry

preventing a linear coupling of S , then order H A-terms can also be eliminated, even in F-term inflation models.

The baryon number from AD baryogenesis is generated at $H \approx m_{susy} \sim 100$ GeV (where m_{susy} is the mass scale of the gravity-mediated soft SUSY breaking terms [21]), when the A-term can introduce B and CP violation into the coherently oscillating AD scalar [4, 5, 22]. If the phase of the AD scalar relative to the real direction (defined by the A-term) is θ , then the baryon number density is (see Appendix)

$$n_B \approx m_{susy} \phi_o^2 \sin 2\theta , \quad (33)$$

where ϕ_o is the amplitude of the coherent oscillations at $H \approx m_{susy}$. Thus

$$\frac{\delta n_B}{n_B} = \frac{2\delta\theta}{\tan(2\theta)} . \quad (34)$$

$\delta\theta$ is generated as usual by quantum fluctuations of the AD field at horizon crossing,

$$\delta\theta \approx \frac{H}{2\pi\phi} , \quad (35)$$

corresponding to fluctuations of the AD scalar orthogonal to the radial direction. Thus

$$\frac{\delta n_B}{n_B} \approx \frac{H}{\pi\phi \tan(2\theta)} . \quad (36)$$

The isocurvature perturbation of the CMB is then given by [2, 23, 24]

$$\alpha = \left| \frac{\delta_\gamma^i}{\delta_\gamma^a} \right| = \frac{\omega}{3} \left(\frac{2M^2 V'(S)}{V(S) \tan(2\theta) \phi} \right) , \quad (37)$$

where δ_γ^i is the perturbation in the photon energy density due to isocurvature perturbations and δ_γ^a is the perturbation due to adiabatic perturbations. For purely baryonic isocurvature perturbations

$$\omega = \frac{\Omega_B}{\Omega_m} , \quad (38)$$

where Ω_B is the ratio of the energy density in baryons to the critical energy density and Ω_m the corresponding ratio for total matter density. For the case of D-term inflation this gives

$$\alpha = \frac{1}{6\pi} \frac{g\omega M}{\phi N^{1/2} \tan(2\theta)} , \quad (39)$$

where $N \approx 50$.

Introducing the upper bound on ϕ from the requirement that the deviations from the spectral index due to the AD scalar are acceptably small then gives, for $d = 4$,

$$\alpha > \alpha_c = \frac{3.3\omega(g\lambda)^{4/9}}{K^{1/9}\text{Tan}(2\theta)} , \quad (40)$$

and for $d = 6$,

$$\alpha > \alpha_c = \frac{0.18\omega(g^3\lambda)^{4/17}}{K^{1/17}\text{Tan}(2\theta)} . \quad (41)$$

The range of Ω_B allowed by nucleosynthesis is $0.006 \lesssim \Omega_B \lesssim 0.036$ [25], where we have taken expansion rate parameter h to be in the range $0.6 \lesssim h \lesssim 0.87$ [26]. Thus, for $\Omega_m = 0.4$ (in keeping with supernova distance measurements [27]) and $K = 0.2$ we obtain for $d = 4$

$$\alpha_c = (0.06 - 0.36) \frac{(g\lambda)^{4/9}}{\text{Tan}(2\theta)} , \quad (42)$$

and for $d = 6$

$$\alpha_c = (3.0 \times 10^{-3} - 0.018) \frac{(g^3\lambda)^{4/17}}{\text{Tan}(2\theta)} . \quad (43)$$

(The lower limits above should be multiplied by 0.4 for the case $\Omega_m = 1$.) Thus if, for example, $g \sim \lambda \sim 0.1$ and $\text{Tan}(2\theta) \lesssim 1$, we would obtain a lower bound $\alpha \gtrsim 10^{-2}$ for $d = 4$ and $\alpha \gtrsim 10^{-3}$ for $d = 6$.

It is interesting to note that present observations of the CMB combined with large-scale structure from cold dark matter (CDM) require that $\alpha \lesssim 0.1$ [23, 24]. In particular, using COBE normalized perturbations combined with the value of σ_8 (the rms of the density field on a scale of 8 Mpc) from X-ray observations of the local cluster together with the value of the shape parameter ($\Gamma \approx \Omega_m h = 0.25 \pm 0.05$ [28]) from the galaxy survey (also consistent with recent observations of high-redshift supernovae [27]), Kanazawa et al [24] conclude that α must be less than 0.07¹. In addition, they show that the COBE-normalized best fit to large-scale structure (σ_8) in a flat Universe with expansion rate parameter $h \approx 0.7$ (in accordance with recent observations)

¹Kanazawa et al define α to be the ratio of the power spectra of the isocurvature to the adiabatic perturbation. This must be multiplied by 16/25 in order to obtain values consistent with our definition of α [29].

is given by $\alpha \approx 0.03 \pm 0.01$. (Large-scale structure cannot be understood on the basis of CDM with adiabatic perturbations alone.) This is exactly in the range expected from minimal $d = 4$ AD baryogenesis in the context of D-term inflation. Thus isocurvature perturbations from AD baryogenesis may already have been observed, although this conclusion very much depends on accepting the COBE normalization.

In any case, future CMB observations by MAP will be able to probe down to $\alpha \approx 0.1$, whilst PLANCK (with CMB polarization measurements) should be able to see isocurvature perturbations as small as 0.04 [29]. Thus for the case of minimal ($d = 4$) AD baryogenesis, if inflation is D-term then there is a good chance that PLANCK will be able to observe isocurvature perturbations. For higher dimension AD baryogenesis ($d \geq 6$) it is less certain, but if ϕ is an order of magnitude below the upper bound from adiabatic perturbations we could still observe the isocurvature perturbations.

All this assumes that ϕ can take any value. This is true if $\phi_i < \phi_*$, in which case ϕ remains at its initial value ϕ_i . However, we have seen that the dynamics of the AD field during D-term inflation implies that if $\phi_i > \phi_*$ then ϕ will equal ϕ_* at $N \approx 50$. In this case we can fix the magnitude of the isocurvature perturbation. For $d = 4$, $N \approx 50$ and $\Omega_m = 0.4$ the magnitude of the isocurvature perturbation is given by,

$$\alpha = \alpha_* \approx (0.17 - 1.03) \left(\frac{N_S}{50} \right)^{1/4} \frac{(g\lambda)^{1/2}}{\tan(2\theta)}. \quad (44)$$

(For $\Omega_m = 1$ this should be multiplied by 0.4.) For $d = 6$ and $\Omega_m = 0.4$,

$$\alpha = \alpha_* \approx (4.4 \times 10^{-3} - 2.6 \times 10^{-2}) \left(\frac{N_S}{50} \right)^{1/8} \frac{g^{3/4} \lambda^{1/4}}{\tan(2\theta)}. \quad (45)$$

If $g, \lambda \gtrsim 0.1$ then for the $d = 4$ case we expect $\alpha_* \approx 0.01 - 0.1$, which is likely to be observable, with the value of the isocurvature perturbation being about three times the lower bound expected from the adiabatic perturbation. For the $d = 6$ case the isocurvature perturbation may just be observable if the baryon asymmetry is close to the upper bound imposed by nucleosynthesis and g, λ and θ take on favourably large and small values respectively.

It is important that we can fix the isocurvature perturbation to be not much larger than the lower bound coming from adiabatic perturbations. This is because there is

typically a very small range of values of ϕ over which the isocurvature perturbation is less than the present observational limit, $\alpha \lesssim 0.1$, but larger than the adiabatic perturbation lower bound, $\alpha \gtrsim 0.01$ for $d = 4$. If ϕ was more than an order of magnitude below its adiabatic upper bound, we would expect to have seen the isocurvature perturbation already, and in general there is no reason for the value of ϕ to be close to the adiabatic upper bound. However, we have shown that the case where S and ϕ have large initial values ("chaotic inflation" initial conditions) provides a natural explanation for a small but potentially observable value of the isocurvature perturbation.

3 F-term Inflation

The results for the case of D-term inflation are based on (i) the absence of order H^2 corrections to the mass squared terms of the AD scalar during inflation and (ii) the absence of order H corrections to the A-terms both during and after inflation. As discussed in the Introduction, models based on F-term inflation must assume that the problem of order H^2 corrections to the mass squared of the inflaton has been solved. In this case we can still have isocurvature perturbations associated with the Affleck-Dine scalar *if* there are no order H corrections to the A-terms, which will be the case if there is a symmetry forbidding a linear coupling of the inflaton superfield to gauge-invariant operators made of MSSM superfields, e.g. a discrete symmetry $S \leftrightarrow -S$ or an R-symmetry. However, in the case of F-term inflation we expect in general that the AD scalar will have an order H^2 mass squared term during inflation. If this correction were positive in sign, the minimum of the potential would be at $\phi = 0$ and the AD field would be damped to be exponentially close to $\phi = 0$ by the end of inflation, preventing AD baryogenesis. Thus the order H^2 correction must be negative. This will *fix* the value of the AD scalar during inflation to be at the $\phi \neq 0$ minimum of its potential, which is essentially fixed by H and d . This in turn will fix the magnitude of the isocurvature perturbation in F-term inflation.

During F-term inflation, the potential of the AD scalar is given by

$$V_{total}(\phi) = -\frac{cH^2\phi^2}{2} + V(\phi) , \quad (46)$$

where $V(\phi)$ is the usual potential from the non-renormalizable superpotential term and $c \approx 1$. The minimum is at

$$\phi_m = \left(\frac{2^{d-2}c}{(d-1)\lambda^2} \right)^{1/(2d-4)} \left(H^2 M^{2(d-3)} \right)^{1/(2d-4)} . \quad (47)$$

Let us first note that if ϕ is close to ϕ_m ($|\delta\phi| \equiv |\phi - \phi_m| \lesssim \phi_m$) then inflation will damp $\delta\phi$ to be close to zero. The equation of motion for perturbations around the minimum is

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} = -kH^2\delta\phi ; \quad k = (2d-4)c \gtrsim 1 . \quad (48)$$

This has the solution

$$\delta\phi = \delta\phi_0 e^{\alpha Ht} ; \quad \alpha = \frac{1}{2}(-3 + \sqrt{9-4k}) . \quad (49)$$

Thus so long as $Ht \gg 1$ i.e. there is a significant number of e-foldings of inflation before $N \approx 50$, the AD field will be damped to be exponentially close to the minimum of its potential.

In general, it is likely that the initial value of ϕ will not be close to ϕ_m . However, we can show that deviation of the adiabatic perturbation from scale-invariance imposes that the value of the potential at $N \approx 50$ cannot be very much larger than ϕ_m . To see this, suppose that ϕ is initially much larger than ϕ_m and consider the contribution of ϕ to the spectral index. During inflation the invariant $\zeta = \delta\rho/(\rho+p)$ is given by $\delta\rho/(\dot{\phi}^2 + \dot{S}^2)$. Since ϕ will not be slow-rolling ($V''(\phi) \gg H^2$) we must have $\dot{S}^2 \gg \dot{\phi}^2$ in order to have a nearly scale-invariant spectrum. We can also assume that $\delta\rho$ comes mostly from quantum perturbations for the S field, as the ϕ field is not effectively massless. Therefore $\zeta \propto (V(\phi) + V(S))^{3/2}/V'(S)$. The deviation from scale-invariance due to the ϕ field is then

$$\Delta n_\phi = -\frac{2}{\xi} \frac{d\xi}{dN} = -\frac{3V'(\phi)}{V(\phi) + V(S)} \frac{\partial\phi}{\partial N} . \quad (50)$$

For $\phi \gg \phi_m$ the ϕ field will be rapidly oscillating in its potential and the change in the amplitude of ϕ over an e-folding due to damping by expansion will be $\partial\phi/\partial N \sim -\phi$. Therefore requiring that $|\Delta n_\phi| < K$ imposes an upper bound on ϕ ,

$$\phi \lesssim \left(\frac{Kd}{6(d-1)\lambda^2} \right)^{\frac{1}{2(d-1)}} \sqrt{2} H^{\frac{1}{d-1}} M^{\frac{d-2}{d-1}}. \quad (51)$$

Thus

$$\frac{\phi}{\phi_m} \lesssim \left(\frac{d}{6} \right)^{\frac{1}{2(d-1)}} \left(\frac{K^{1/(2d-2)}}{c^{1/(2d-4)}} \right) \left(\frac{\sqrt{d-1}\lambda M}{H} \right)^{\frac{1}{(d-1)(d-2)}}. \quad (52)$$

For $d = 4$,

$$\frac{\phi}{\phi_m} \lesssim \frac{0.8}{c^{1/4}} \left(\frac{\lambda M}{H} \right)^{\frac{1}{6}}, \quad (53)$$

whilst for $d = 6$

$$\frac{\phi}{\phi_m} \lesssim \frac{0.9}{c^{1/8}} \left(\frac{\lambda M}{H} \right)^{\frac{1}{20}}, \quad (54)$$

where we have used $K = 0.2$. Therefore for typical values of H during inflation, scale-invariance of the density perturbations implies that ϕ at $N \approx 50$ cannot be much more than an order of magnitude greater than ϕ_m . Since there is no reason for ϕ to be close to this upper limit when $N \approx 50$, it is most likely that ϕ will be close to ϕ_m when the CBR is formed.

Given that $\phi \approx \phi_m$, the isocurvature perturbation is given by

$$\alpha \approx \frac{2\omega}{3} \frac{H}{Tan(2\theta)\delta_\rho\phi_m}, \quad (55)$$

where $\delta_\rho = 3\delta T/T \approx 3 \times 10^{-5}$ is the value of the CBR energy density perturbation [1]. Therefore given H and d , the value of ϕ_m and so the magnitude of the isocurvature perturbation is essentially fixed. For $d = 4$ and $\Omega_m = 0.4$ we find

$$\alpha = (3.1 - 18.6) \times 10^2 \frac{\lambda^{1/2}}{c^{1/4}Tan(2\theta)} \left(\frac{H}{M} \right)^{1/2} \quad (56)$$

whilst for $d = 6$

$$\alpha = (2.9 - 17.4) \times 10^2 \frac{\lambda^{1/4}}{c^{1/8}Tan(2\theta)} \left(\frac{H}{M} \right)^{3/4}. \quad (57)$$

If we require that $\alpha \lesssim 0.1$ in order that the isocurvature perturbation has not been observed at present, these impose upper bounds $H/M \lesssim 10^{-7}/\lambda$ (for $d = 4$) and $H/M \lesssim 10^{-5}/\lambda^{1/3}$ (for $d = 6$). Thus for typical values of H the isocurvature perturbation in the F-term inflation case can be close to present observational limits.

4 Conclusions

We have considered the dynamics of an Affleck-Dine scalar in the MSSM in the context of D- and F-term inflation models and the associated adiabatic and baryonic isocurvature perturbations. In the case of D-term inflation, if the AD scalar is initially large (as one would expect if the fields obeyed chaotic inflation-like initial conditions) then ϕ at the time when the CMB goes beyond the horizon will be essentially fixed, with a weak dependence on the total number of e-foldings of inflation. In this case we can predict the magnitude of the isocurvature perturbation. For $d = 4$ AD baryogenesis this will be typically in the range $\alpha = 0.01 - 0.1$ and is likely to be observable by PLANCK. This is also consistent with the value $\alpha = 0.03 \pm 0.01$ for which a mixed adiabatic and isocurvature perturbation spectrum can account for large scale structure observations of σ_8 , the shape parameter Γ and the present expansion rate, $h \approx 0.7$ (the latter implies from Γ that $\Omega_M \approx 0.4$, consistent with observations of Type Ia supernovae), which cannot be understood on the basis of adiabatic perturbations alone. Therefore isocurvature fluctuations from D-term inflation/ $d = 4$ AD baryogenesis may already have been indirectly observed.

More generally, deviation of the adiabatic perturbation from scale-invariance due to the AD scalar imposes an upper bound on the magnitude of the AD scalar, which in turn imposes a lower bound on the isocurvature perturbation. For $d = 4$ AD baryogenesis, the lower bound on α is greater than 0.01 for typical values of the unknown parameters, again suggesting that the isocurvature perturbation can influence large-scale structure formation and is likely to be observable by PLANCK. To find precisely the expected limit one should perform a simultaneous fit of all the relevant cosmological parameters to the simulated data. One should take properly into account the

correlation between adiabatic and isocurvature perturbations, as well as the degeneracy between isocurvature and tensor perturbations, which can be resolved by the polarisation data [29]. We should also like to point out the AD isocurvature fluctuations are not gaussian, a fact which can be used to further constrain the amplitudes and hence AD baryogenesis.

In the case of F-term inflation, the value of the AD scalar when the CMB goes beyond the horizon will most likely be at the minimum of its potential, as determined by the negative order H^2 correction to its mass squared term. Thus if there is an isocurvature perturbation (which is possible if there are no order H corrections to the A-terms, which simply requires that there is no linear coupling of the S superfield to MSSM fields), its magnitude will be fixed by d and the value of H during inflation. For $d = 4$ ($d = 6$) AD baryogenesis, $H \lesssim 10^{-6}$ (10^{-4}) is necessary for the isocurvature perturbations to be consistent with current observations ($\alpha \lesssim 0.1$). For reasonable values of H the isocurvature perturbations can be large enough to be observable by PLANCK, although, unlike the case of D-term inflation, there is no strong reason to expect observable perturbations.

We previously discussed the D-term inflation case for $d = 6$ AD baryogenesis with the formation of late decaying Q-balls of baryon number [30]. This variant of AD baryogenesis, "B-ball Baryogenesis" [30, 31, 18], is a natural possibility in the MSSM. In this case the baryonic isocurvature perturbations of $d = 6$ AD baryogenesis are amplified by being transferred to dark matter neutralinos via late decay of the B-balls, and are naturally in the observable range. So observation of isocurvature perturbations by PLANCK, combined with the observation of a deviation of the adiabatic perturbation from scale invariance as predicted by D-term inflation, would indicate in the context of the MSSM either $d = 4$ AD baryogenesis with conventional thermal relic neutralino dark matter [32] or $d = 6$ AD baryogenesis with non-thermal neutralino dark matter from late-decaying B-balls [31, 33].

Clearly the observation of isocurvature perturbations by PLANCK, together with a deviation of the density perturbations from scale-invariance consistent with D-term inflation, would have profound implications for both inflation and origin of the baryon

asymmetry. Indeed, the fact that the expected magnitude of the isocurvature perturbations from $d = 4$ AD baryogenesis is consistent with present observations of large-scale structure may already be indirectly telling us something fundamental about the nature of inflation and the baryon asymmetry, which hopefully will be clarified by direct observations of the density perturbations by PLANCK. At the very least, some forms of AD baryogenesis can be ruled out by the forthcoming CMB observations.

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Appendix. Affleck-Dine Baryogenesis

The full scalar potential along a flat direction of the MSSM in the early Universe has the form [5, 11]

$$V(\Phi) \approx (m_{susy}^2 - cH^2)|\Phi|^2 + \frac{\lambda^2|\Phi|^{2(d-1)}}{M_p^{2(d-3)}} + \left(\frac{A_\lambda \lambda \Phi^d}{dM_p^{d-3}} + h.c. \right), \quad (58)$$

where m_{susy} is the gravity-mediated SUSY breaking mass term, typically of the order of 100 GeV. In both D- and F-term inflation, once inflation ends and the inflaton begins to coherently oscillate about the minimum of its potential the AD scalar will have an order H^2 correction to its mass squared term. (In D-term inflation this is because F_S is non-zero when $\dot{S} \neq 0$ [9].) In order to have an unsuppressed value of ϕ at $H \approx m_{susy}$, the order H^2 correction should be negative. (In fact, in D-term inflation models, for $|c|$ less than about 0.5 it is possible to have a positive H^2 correction and still generate the observed baryon asymmetry [34]. Here we will concentrate on the negative H^2 correction.) The AD scalar sits at the minimum of its potential until $H \approx m_{susy}$, at which time its mass squared term becomes dominated by the gravity-mediated term and changes sign and the AD scalar begins to coherently oscillate about its new minimum at zero. The A-term is dependent upon the phase of the AD field

and so can induce B and CP violation in the coherently oscillating AD field. In the absence of order H corrections to the A-terms, the initial phase θ of the AD field (relative to the real direction as defined by the A-term) is random and so typically ≈ 1 . When the AD field starts to oscillate at $H \approx m_{susy}$, the A-term is of the same order of magnitude as the mass squared term, and so the A-term will cause the mass of the scalars along the real and imaginary direction to differ by $O(m_{susy})$. As a result, these will oscillate with a phase difference $\delta \approx 1$. After a few expansion times, the amplitude of the oscillations will become damped by the expansion of the Universe and the A-term, which is proportional to a large power of ϕ , will become negligible, so fixing the B asymmetry in the AD condensate. The B asymmetry is given by

$$n_B = i(\dot{\Phi}^\dagger \Phi - \Phi^\dagger \dot{\Phi}) . \quad (59)$$

With $\Phi = (\phi_1 + i\phi_2)/\sqrt{2}$, where $\phi_1 = \phi_o \cos(\theta) \sin(m_{susy}t)$ and $\phi_2 = \phi_o \sin(\theta) \sin(m_{susy}t + \delta)$, the baryon asymmetry is therefore given by

$$n_B \approx \frac{m_{susy} \phi_o^2}{2} \sin 2\theta \sin \delta . \quad (60)$$

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